On Teaching and Learning

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On Teaching and Learning publishes articles and essays on aspects of pedagogical practice and on research that has implications for teaching.

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Transforming Mathematics Pedagogy
Pat Rogers

Listen to a woman groping for language in which to express what is on her mind, sensing that the terms of academic discourse are not her language, trying to cut down her thought to the dimensions of a discourse not intended for her.

Introduction
A forceful impetus for changing my teaching has been my own experiences in undergraduate mathematics and doing mathematical research. My thinking gained momentum during my visit to the Mathematics Department of the State University of New York at Potsdam, where my research has shown that it is possible for women to excel in the study of mathematics with male teachers without the negative consequences so often associated with learning in a patriarchal environment. When the style of teaching is true to the nature of mathematical inquiry, and when the environment is genuinely open to and supportive of all students, women are attracted to mathematics and are just as successful as men (Rogers 1988, 1990).

Disempowerment
When I first started teaching, I used those practices I had observed as a student: I lectured. I believed that teaching at the post-secondary level involved the transmission of knowledge from me, the “expert,” to the students, the “novices.” I saw my job as “exposing” my students to the material of the course. The student’s job was to “master” this material by listening attentively, by watching me model how to solve problems, and by practicing solving problems on their own at home. I administered tests in order to measure achievement and to rank students in relation to their peers. Each class had a natural pattern: I introduced the topic, covered the blackboard with formulas and mathematical language, and worked a few problems. I asked a few questions, and even elicited a few answers, though usually from the same 3 or 4 (male) students, and then I assigned homework. I was considered a successful teacher. In my course evaluations, students praised me for my enthusiasm, my organization, the clarity of my exposition, my knowledge of the material, and my accessibility. The most critical comment was a request to slow down a little. Yet, on the final examination many students failed or wrote such incomprehensible answers that I wondered if we had all been engaged in the same course. If I had explained the material so well, how could they do so badly? How could they not buy the goods I had sold them so persuasively? Of course this troubled me, but it was easy enough to dismiss: “Students come to university so ill-prepared. If only we had better students, just think what we could do!” A familiar refrain.

Over the years I became increasingly concerned about the students in my class (mainly female) who never spoke a word. These were students who sorely needed individual attention yet never used my office hours. They were the students who were heading for certain failure but were lulled into thinking they might pass the course because, as they often said, “It seems so easy when you do it on the board.” I only began to hear the hidden messages my students were sending me when I began to reflect upon my own experiences as a mathematics undergraduate at Oxford. It shocked me to realize how faithfully I was reproducing in my own classroom the structures which had so effectively silenced and disempowered me at that time.
Whose mathematics is taught?

As a high school student, I found that I was able to recon­struct for myself any of the formulas, rules or results that I needed. Some of my teachers even admitted to not knowing answers to many of problems that I posed. However, at the university the professor was distant, remote and expert, and I began to experience mathematics as something distant and imposed, not something that I had a role in producing. We were rarely given the opportunity to play with mathematical ideas or to construct our own meanings (except on our own at home). Instead, through the medium of the polished lecture (or textbook), mathematics came to me finished, absolute and pre-digested.

A pedagogy which emphasizes product deprivs students of the experience of the process by which ideas in mathematics come to be. It perpetuates a view of mathematics in which right answers are the exclusive and sole property of experts. Such a pedagogy strips mathematics of the context in which it was created and reinforces misconceptions about its very nature. Students permitted to see only the polished product may come to believe they can never create similar results for themselves (“I don’t know where to begin!”).

Carol Gilligan’s research in the field of women’s moral development provides a link between mathematics avoidance and mathematics pedagogy. She identified two styles of reasoning, which although not gender-specific are thought to be gender-related: one, the traditional style, is characterized by objectivity, reason, logic, and an appeal to justice; and the other, “the different voice,” often identified with women and as a consequence devalued, is characterized by subjectivity, intuition, and a desire to maintain relationships. The “different voice” is rooted in an ethics of care and responsibility.

Dorothy Buerk has studied Gilligan’s styles in relation to the work of mathematicians. A group of mathematicians was presented with examples of “separate” reasoning (the traditional style) and with “connected” reasoning (the different voice). They unanimously identified the “connected” exam­ples as representing the way mathematicians do mathematics. “[M]athematics is intuitive,” they said. They stressed the creative side: attention to the limitations and exceptions to theories, the connections between ideas, and the search for differences among theories and patterns that appear similar. And yet they agreed that the ‘separate’ list conveyed the way that mathematics is communicated in the classroom, in textbooks, and in their professional writing” (Buerk, 1985).

In reality, mathematicians employ both forms of reasoning in their work. But the problem with mathematics teaching, particularly at the post-secondary level where the lecture mode of instruction is so predominant, is that the creative-intuitive form is largely eliminated. Students are not given the opportunity to become involved in the process of constructing mathematical ideas, a process in which “connected” thought is so important. There is an enormous cognitive gulf between the way mathematics is presented in textbooks and the individual ways in which it is possible and natural to arrive at an understanding of it. Some students are able to bridge this gap for themselves, but many are not.

The expository approach to teaching mathematics affects all students, but does not affect them all to an equal extent. If Gilligan’s claim that women favor a “connected” reasoning style has validity, then the tendency most teachers have of utilizing only the traditional “separate” route to mathematics could inhibit or prevent some students — especially women students — from claiming mathematics for themselves.

How is mathematics taught?

One important component of the expository method of teaching is the use of the distant authority to “impart knowl­edge.” Practices such as lecturing (or preaching) subordinate students’ knowledge and understanding to that of the profes­sor and the even more distant authority, the textbook. Solving problems for students does not teach them to solve problems for themselves. Instead, it disempowers students by rendering them passive, and conveys mistaken notions such as, “there is
A mathematics pedagogy of possibility

I would like to outline an alternative approach to the teaching of mathematics, one which is rooted not in the authoritative and imposed style which distances and silences, but in a style which encourages direct access and engagement, free creative expression, and ownership of subject. This alternative might be thought of as introducing the element of the "feminine."

The first element of this approach has been influenced, among other things, by an ethics of caring and responsibility that is designed to enable those who have been silenced to speak (Simon, 1987). One must become, first and foremost, a "caring teacher." Here I use the term caring not simply as a feeling of concern, attentiveness, or solicitude for another person, but in the very specific sense of helping "the other grow and actualize himself" (Mayeroff, 1971). The caring teachers I have observed at Potsdam possess the ability to focus primarily, or at least initially, upon the student rather than upon the subject matter, with the idea that the route to a subject is not imposed from without, but rather illuminated from within. These instructors do not work upon their students, but with their students, looking at the subject matter from their perspective and at their level. It is a student-sensitive pedagogy (Rogers, 1988), grounded in the student's own language, focused on process rather than on content, and centered in the student's individual questions and learning processes. Students who are "cared for" in this way are set free to pursue their own legitimate projects (Noddings, 1984).

Another element in this approach involves demystifying the doing of mathematics. This includes such things as calling students' attention to mathematics as a creation of the human mind, making visible the means by which mathematical ideas come into being and the process by which they are polished for public consumption. It also includes encouraging students to challenge authoritative discourse, and to feel that the gates to the mathematical community are open and that they have the skills they need to walk through and to operate within it.
Reflecting on the differences between what I was doing in my early teaching and what I came to believe needed to be done, I realized that a core difference lay in the engagement of students within the classroom in purposeful, meaningful activity.

Putting theory into practice

I would like to describe some of the ways I put the ideas outlined above into practice. At the beginning of a course I discuss my goals with the students and make the goals available in written form. They fall into three categories: content-specific goals, process goals, and communication/social goals. The content-specific goals include the development of students' skills in constructing their own proofs of mathematical statements and writing them clearly and in correct form. Process goals include the development of independent working skills (the more highly developed the individual working skills, the less the need for a teacher), such as reading a mathematics text with understanding, finding, analyzing and correcting mistakes, and asking mathematical questions. Since mathematics is an activity which depends on communication between colleagues, social goals are also stressed: including helping students collaborate with others; communicate ideas clearly and with confidence both orally and in writing; listen actively; offer constructive criticism; and ask and respond to challenging questions. The classroom teaching methods I use vary depending on the characteristics of the particular class and the questions raised by the students. Following is a brief description of some of the methods I use most frequently.

Lecturing. I have not abandoned lectures; instead I use them sparingly for three main purposes: to introduce a new section of material; to conclude a topic and draw everything together; or to introduce a new concept and motivate the reading of assignments (this is usually a short lecture given at the end of a class).

Think-write-pair-share. The “think-write-pair-share” idea is adapted from Davidson et al. (1986) and is the most useful and most frequently used of my current strategies. It is also the one that is most adaptable to classes of all sizes. In class students are presented with a question, or given a segment of the text to read. They work independently at first, put their thoughts and ideas down in writing, and then form pairs to discuss. This provides support for those students who are unsure of their ideas or who have a fear of appearing foolish before the class. It has the effect of increasing participation and involving all students in the affairs of the class. This may precede or be integrated into all the activities which follow.

Full class dialogue. Like a lecture, full class dialogue is teacher-directed. Unlike a lecture, it is student-centered. Dialogue usually follows assigned reading and think-write-pair-share activity, and operates by way of questioning. My questions aim to help students express their ideas clearly and with precision. For this, good active listening skills on my part are essential: I demand reasons for statements, and challenge students to formulate their ideas in their own words and explain them to each other. The latter is important because students are often unable to “hear” their instructor, but are perfectly able to hear one another, and such listening and telling is empowering.

Round robin. I ask questions of each student in turn, giving students the right to “pass.”

Boardwork. Students come to the board individually or in small groups to write up questions, problems, and solutions. We then fully discuss the solutions. I may assign problems ahead of time, and I may allow time in class for students to work on solutions in groups prior to the boardwork. Sometimes several groups will work the same problem at the board. I can then demonstrate that there is no one right approach to solving a problem. Students learn from each other’s mistakes and learn both oral and written presentation skills. At first, I do not require that students remain at the board to defend their
solution. However, as the classroom climate becomes more supportive and students begin to encourage each other to participate, confidence in their ability to discuss mathematical ideas increases.

**Brainstorming.** I give student pairs from 2-5 minutes to write down everything they know about a given topic. I then call on the pairs and all the information is put on the blackboard. Discussion then centers around categorizing and evaluating what has been gathered. I use this technique most often for review or to begin an **investigative class** (see below).

**Problem-posing.** Students generate their own questions in a particular area. These are then examined, and the students commit themselves to a particular conjecture. For example, this year an early conjecture was Lagrange’s theorem. Proving this theorem became the focus of attention over several weeks. This focus made the course problem-driven, a response to the students’ own and best avenue of inquiry. An activity of this sort usually occurs naturally, unlike the investigative class to follow, which is artificially established.

**Investigative class.** The class examines patterns found in concrete examples. The purpose is to uncover algebraic structure. Generalizations are then made in the form of conjectures, and a theory which proves or disproves the conjectures is developed. I have used investigative classes to generate all standard structure theorems for finite groups.

**Small group work.** Groups of two to four students work together while I circulate among them and check their work. This provides me with immediate feedback on student understanding and enables students to influence the pace and the development of the course. Even in a large class, where I could check only the work of students sitting at the aisle, I was provided with immediate feedback. Students in that class accused me of checking up on them to see if they were doing the homework. I asked them whether they would prefer me to wait for the test before I found out how much they understood, and I pointed out that a winning strategy for test-taking might be to always sit near the aisle. This produced a very curious seating arrangement for the remainder of the classes. The center of the room emptied out, and all students tried to sit near the aisle!

**Reading Exercises.** The development of good reading skills is essential to gaining independence in mathematics. The following exercises are quite effective:

(a) A proof is assigned (see figure 1). Question marks are inserted at crucial junctures in the argument. Students rewrite the proof, replacing each question mark with an explanation.

(b) A proof is assigned, as in (a), but students have to insert their own question marks indicating where they question what is given.

(c) A proof is assigned, as in (a), and question marks are explained orally in class. This exercise also develops students’ communication skills.

(d) A definition for reading is assigned from the text. Students, usually in pairs, construct their own meaning of the definition by working through an assigned exercise.

(e) Students are asked to pull apart the statement of a theorem and write it in their own words before attempting to prove the result. All technical words must be defined. The structure of the statement is examined and a proof strategy outlined. This is often done in conjunction with the forward-backward proof technique (see below).
Theorem: R/A is a Field if and only if A is Maximal

Let R be a commutative ring with unity and A an ideal of R. Then R/A is a field if and only if A is maximal.

Proof: Suppose R/A is a field and B is an ideal of R that properly contains A. Let b ∈ B, but b ∈ A. Then b + A is a nonzero element of R/A, and therefore, there exists an element c + A such that (b + A)(c + A) = 1 + A, where 1 + A is the multiplicative identity of R/A. Since b ∈ B, we have bc ∈ B. Because

\[ 1 + A = (b + A)(c + A) = bc + A, \]

we have 1 - bc ∈ A. So, 1 = (1 - bc) + bc ∈ B. Thus, B = R. This proves that A is maximal.

Now suppose that A is maximal and let b ∈ R but b ∈ A. It suffices to show that b + A has a multiplicative inverse. Consider B = {br + a | r ∈ R, a ∈ A}. This is an ideal of R that properly contains A. Since A is maximal we must have B = R. Thus, 1 ∈ B. So, 1 = bc + a' where c ∈ R and a' ∈ A. Then,

\[ 1 + A = bc + a' + A = bc + A = (b + A)(c + A). \]

Proof generation: The ‘forward-backward proof technique’ (see Figure 2 for an example) described by Solow (1982) in his book, How to Read and Do Proofs, is a very successful technique. It takes its name from the way mathematicians typically organize their thoughts when constructing the proof of a statement for the first time. Although it cannot be successfully applied in all situations, it has virtually eliminated students’ complaints that they “don’t know how to begin!”

Euclid’s Lemma

If p is a prime that divides ab, then p divides a or p divides b.

Proof in construction: First we untangle the statement of the theorem and decide on the structure of the proof. The statement has the form “if...then.” This means that we may begin by assuming that p | ab and must prove that p | a or p | b. An “either/or” statement can often be proved by assuming that one case does not occur and then showing that the other must occur. In this case then we may assume that p | ab, p | a, and try to prove that p | b.

Forward: Assume p | ab and p | a.

\[ p | ab \text{ implies there exists an integer } k \text{ such that } ab = kb. \]

Since p | a and p is a prime number, p and a must be co-prime, i.e., (p, a) = 1.

So, there are integers s and t such that sp + at = 1.

Thus we have:

\[ ab = kb \]
\[ sp + ta = 1 \]

for some integers k, s, and t.

STUCK — look at what we have to prove.

Backward: We need to prove that p | b, i.e. we need to find an integer m such that

\[ b = mp. \]

(The questions I ask that will often get the students moving in a case like this are: if p | b were the last line of the completed proof, what could the line before it look like? What might you have proved that would convince me your conclusion was correct?)
STUCK — check where we left off to see whether this helps.

Forward: We must express $b$ as a multiple of $p$. (If you need something take it!)

Multiplying both sides of (2) by $b$ gives

$b (sp + ta) = b,$

which can be rearranged as

$b = bsp + bta ....................(4)$

STUCK

Backward: Looking back again at what we are trying to prove reminds us that we need to isolate $p$ as a factor on the right side of equation (4). Only the term $bta$ is not already in the required form. What do I know that will help? What haven't I used yet? Checking over the forward direction of the proof, we see that equation (1) has not been used.

Forward: From equation (1), $ab = kp$, thus $bta = tab = tkp$. Substituting for $bta$ in equation (4) gives

$b = bsp + bta = bsp + tkp = (bs + tk)p$. Since $b, s, t,$ and $k$ are all integers, this expresses $b$ in the required form $mp$, where $m = bs + tk$ is an integer. This proves that $p | b$.

The final task I give students is to polish the proof for "publication," i.e., write their own version of the proof in the usual paragraph form.

Evaluation of Student Learning

Evaluation of student learning is ongoing in the sense that the student-centered techniques I employ provide immediate, frequent and regular feedback on students' understanding of the course content and processes. Following is a brief description of some of the more formal evaluation methods I employ:

Assignments. I assign homework regularly throughout the course and space the assignments so that they are combined with quizzes and examinations, giving regular feedback on three kinds of written work. I encourage students to collaborate on assignments but will accept only independent write-ups. I comment on the homework but do not grade it. The purpose of these assignments is to allow students to practice freely with the concepts and processes of the course without being penalized for doing so.

Participation. Students can earn participation credit in a variety of ways, allowing for their individual learning preferences. Some of these ways include visiting me for an office consultation, presenting aspects of assigned reading in class and working problems from the text, participating in class-assigned exercises, sharing ideas by coming to the blackboard and presenting a proof, asking questions, offering explanations, or joining in discussions. Incidentally, I have found that my office hours are used less extensively than they were before. Students tell me this is because most of their questions...
are either answered in class or by other students. As a student once told me somewhat apologetically: "We don't really need you any more!"

The issue of participation credit in the final grade calculation is a matter of some concern to me. I have used it often in the past and have found that it can foster competition, regardless of the classroom climate I have attempted to create. It may also be a disadvantage to some students. One student complained in the final course evaluation that there had been "too much emphasis on class participation and working on the board. It is uncomfortable for some people." For these reasons, I now use participation, including observable improvement over time, to adjust grades upwards if a student's grade is borderline.

**Quizzes.** I give in-class quizzes regularly. For the first two quizzes, students are given the option of submitting an error analysis (see figure 3). This error analysis provides students with a second chance, an opportunity to improve not only their grade but also their comprehension of the course material. The initial grading is fast; I simply indicate where the student has made a mistake and award a mark, with no explanation. Occasionally, I ask a question calling for clarification of a point in the student's argument. Grading of the error analyses is much more time-consuming. The importance of having students do an error analysis far outweighs its value in increasing a student's grade. An early failure in a course can be very dispiriting for a student. Also, a test should be a vehicle for learning, not solely an instrument of evaluation. Many students tell me that they appreciate the opportunity to learn from their mistakes without being penalized for making them. I have also found that it improves the learning environment in the class and reduces students' test anxiety.

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**Figure 3**

**Guidelines for Doing an Error Analysis**

An error analysis is a second chance, an opportunity to improve your performance on the test and to learn from the experience. All students who received a less than perfect initial test score are encouraged to submit an error analysis.

1. Only look at those questions for which you did not receive a perfect score. This includes questions you did not answer for whatever reason.

2. Start the error analysis by writing out a full, correct answer to the question. This solution can be got by consulting with a friend, by studying solutions to similar problems done in class or in the course texts or, as a last resort, by consulting with me.

3. Directly below your correct answer, explain what your mistake was. Be specific and clear. Don't say "I didn't understand the question." Instead convince me that you know where you went wrong and why. Was it due to unclear thinking? Misreading the question? Copying down an incorrect number? Using the wrong formula or equation? A technical error? An omission? etc.? Where possible, indicate the exact line or step in your solution where things start to go wrong.

4. In a case where you gave an incorrect answer, try to formulate a related question to which your answer is correct.

5. Include a copy of your original test at the back of your error analysis.

6. Your test will be regraded and your final test score will be the average of the "before-error analysis" and "after-error analysis" scores.
Student participation in setting the final examination. I believe in formal examinations. A final performance is an opportunity for students to pull the course together, to see it as a whole and to demonstrate how much they have learned. Last year, I experimented with making students responsible for generating final examination questions. About two weeks before the end of term, time was devoted in class to reflecting in small groups on the course content and brainstorming areas for examination. Four areas were identified, with students indicating the areas which interested them most and dividing themselves up amongst the four areas. Each group was given one week to design two examination questions in its area. Another class was devoted to refining these questions which were then distributed to all students. In the final class, four study groups were formed containing at least one student from each topic area. This meant that each study group contained at least one “expert” in each exam area. This format is known as the jigsaw classroom (Aronson et al, 1978). In constructing the final examination, 50% of the examination questions were chosen from the student question pool. The result of this experiment in collaborative examination design was that it fostered a cooperative spirit among the students who, independently of me, organized a full day of study to prepare themselves for the final examination. York is a large urban commuter university, so this is not a typical response to exam preparation.

Conclusion

I first employed participatory teaching methods in a second year finite mathematics course for non-majors. It was a multi-section course, and my section had 60 students enrolled with no attrition. The two other sections were slightly smaller and both instructors employed a lecture approach exclusively. I called my approach “coaching.” My main objective was to prove to my colleagues that I could teach 60 students without lecturing, without compromising standards, and without dis-advantage to the students either by less coverage of the material or lower success or achievement ratings. The result was encouraging and convincing.

Measured by a common final examination, the three sections of the course had almost identical class averages. However students in my section obtained more A grades overall (27% compared with 22% and 19%); and only 15% of students in my section failed the examination, compared to 24% and 30% who failed in the other two sections. Covering the course content presented no difficulty, either for me or for the students, judging by their examination success. Over two thirds of the students participated actively in the course in one way or another and only one student complained of discomfort.

Judged by examination criteria alone, one cannot conclude that the lecture method came out second best. However, it is clear that in this first try, my “coaching method” held up well against the traditional approach. On the other hand, it is questionable whether the impact of participatory, democratic teaching methods can readily be evaluated in the short-term, or by quantitative means. I was in fact more interested in discovering whether I could teach this way and whether students would be responsive to these methods and learn from them. This took precedence in my mind when comparing these techniques with conventional methods or when trying to prove their superiority.

I would now like to return to the project with which I began this discussion, that of developing a pedagogy designed among other things to accommodate women’s style of learning. For the past two years, I have been refining my approach to participatory teaching in a small, third year algebra course for majors (Groups, Rings, Fields). While it is still too early to make any definitive conclusions, some observations on my experiences with this course are illuminating. During the period I taught the course, student enrollment increased by almost 70%, from 16 to 27, with females outnumbering males 3 to 2. Overall, female students obtained higher final grades than their male colleagues: 31% of all female stu-
Students were in the A range, compared with 24% of the males; no female student obtained a grade lower than C, compared with 6% of the male students who were in the D range. It might be worth noting that this pattern of female-male achievement is remarkably similar to the pattern I observed in the student achievement between my section and the other two sections in the finite math course, discussed above. Attrition in both years was negligible, with a slightly higher percentage of male drop-outs than female (2 out of 17 males dropped the course compared with 2 out of 26 females).

One consequence of teaching in this way, a consequence I have found more rewarding and compelling than any other, is the strong sense of community and caring that develops among students. Students from both years of this course were more concerned about each other’s welfare and progress than anxious to compete with each other for higher grades. Students developed close bonds of loyalty to one another which carried well beyond the course itself and have been long-lasting. Lack of competition is not usually associated with the mathematics classroom (however, see Rogers, 1988, for a description of the Potsdam College mathematics classroom); yet it appears to be a very natural outcome of my teaching now, and evidently provides a more comfortable and equitable classroom environment for the female students I teach.

Now that I know that I can teach this way, I discover that I prefer teaching this way. It allows me to know the students well and I am now much more aware how they comprehend the material — I do not have to wait until the final examination to find that out, as is so often the case when one lectures. By providing opportunities for students to hear and to develop their own voices through engagement in authentic mathematical activity within the classroom, I am able to engage them in purposeful, meaningful academic discourse, allowing them to claim ownership of mathematics for themselves. In so doing, I believe I not only avoid discriminating against students who are currently denied access to mathematics (especially women), but I also provide a more meaningful and equal mathematics education for all students.

Notes

1 I wish fully to acknowledge my indebtedness to the faculty and students of the Potsdam Mathematics Department for allowing me to observe them teaching and learning, as well as to the Social Sciences and Humanities Research Council of Canada for the financial support of this study. Parts of this paper have already been published in different forms (Rogers 1988, 1990).

2 This is not to argue against affirmative action in hiring procedures. However, in the current situation where most mathematics departments are dominated by male faculty members, it is useful to learn of environments which do not inhibit the participation of women.

3 I use this term because it is popular although I would prefer to say that society has chosen to exclude women from the study of mathematics, rather than say that women choose to avoid it.


References


Turn Back, O Man, Forswear Thy Foolish Ways!

Arthur L. Loeb

Would man but wake from out his haunted sleep...

Prior to their retreat in the autumn of 1990, the trustees of Radcliffe College were sent a series of questions, the answers to which we interpreted to yield a personality profile of each trustee. The sequence of the questions was scrambled, but the results were indexed on the basis of four pairs of mutually antithetical parameters. For each of these paired parameters the answers to the list of questions were evaluated on a scale ranging from minus 5 to plus 5.

One of the four pairs was Introversion/Extroversion. Here I scored right in the middle, neither particularly extroverted, nor markedly introverted. This result did not especially surprise me, but it took me back a great many years, to my enrollment as a Freshman, just after arriving in this country on my seventeenth birthday. As part of our matriculation we were required to fill out a questionnaire; little did I suspect that this would be but my first participation in that national pastime of filling out forms. This earlier questionnaire did not beat around the bush: it bluntly asked whether I considered myself an introvert or an extrovert.

I had been reared in my native Holland on the premise that one should not make oneself conspicuous, and that friendships should not be entered into lightly since they were to last not just for a lifetime but for generations to come. Introversion rather than extroversion had been the approved norm. Furthermore, I found myself somewhat overwhelmed

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