On Teaching and Learning

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Price (1990) described, to discover connections between seemingly unrelated concepts. This reexamination is certainly indicative of “writing to learn” within the curriculum.

Notes

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References


Why Women (and Men) Give Up On Science

David Layzer

Why do women turn away from the natural sciences, and especially the physical sciences, earlier and in greater numbers than men? And what should be done to reverse this trend? Discussions of these questions often focus on social attitudes. Parents encourage their sons' interests in things mechanical, electrical, and chemical; they buy dolls for their daughters. In high school a girl who dreams of becoming a theoretical physicist may get less support from parents, peers, and teachers than the male classmates she outperforms in her math and physics classes. Even if she rejects the prevailing belief that women aren't as good at math and physics as men (those who seem to be are over-achievers), she can't wholly ignore it. Probably she is more susceptible to social pressure than her male classmates. So when more rewarding options turn up she is more likely to drop out of math and science than a male student of comparable ability and achievement.

Social attitudes are unquestionably important. But another aspect of the problem has received less attention than it deserves. Many students who fall in love with science and math at an early age fall out of love with them in high school or college. Math and science stop being fun and become boring. Challenge degenerates into frustration. Sheila Tobias makes this point eloquently and persuasively in a recent study that should be required reading for all math and science educators.1

1 DAVID LAYZER is Donald H. Menzel Professor of Astrophysics at Harvard, and has for many years taught in the general education Core Program. Recently, with Professor Dudley Herschbach, he has developed a new introductory course in physics and chemistry which puts into practice a number of the precepts discussed in this paper.
But surely, you may say, the authors and publishers of math and science textbooks, and the teachers who teach from these books, are already working very hard to make their subjects more interesting. Every year the best-selling textbooks for freshman physics, chemistry, and calculus get bigger, glossier, more colorful, more crammed with up-to-the-minute examples and applications. But as these books have become more appealing to the eye, they’ve become less appealing to the inquiring mind. Text is cut to make room for pictures. Diagrams are enlivened with artistic effects and cartoon characters. Algorithms are spelled out and illustrated in ever greater detail. Arguments are simplified and abridged, subtleties are ignored entirely. Worst of all, most modern textbooks in freshman physics, chemistry, or calculus aren’t intended to be read; they’re intended to be consulted. Instead of developing themes and ideas coherently, they offer bit-sized packets of information, exercises, and worked examples.

What has happened to introductory college-level math and science textbooks in part reflects a general decline of educational standards in American schools. College freshmen are, for and large, less skilled in the uses of words and numbers than they were a generation ago. They rely more heavily on pictures than on the written word, and they haven’t been as well trained to recognize or make sound arguments. But the best-selling introductory college-level math and science textbooks not only simplify their subject matter. They also misrepresent it in ways that repel the brightest and most curious students. The law of supply and demand suggests that the courses that use these books — or at least those that rely heavily on them — are similarly repelling.

Sheila Tobias has suggested that there are two tiers of math and science students. Those students who eventually pursue careers in math and science belong mainly to the first tier. By the time these students reach college, their outlook, work habits, and even some of their skills resemble those of young professionals in their fields. In this respect they’re like first-year students at a conservatory. Students in the second tier do not, with rare exceptions, pursue careers in math and science, though many of them go on to medical school. They drop out of math and science or take only the courses they must take to satisfy requirements. Their views of math and science differ strikingly from those of their fellow students in the first tier. Yet they are just as bright as first-tier students, and many of them once dreamed of careers in scientific research. What makes the difference in these two types of students?

Some first-tier students are the sons and daughters of mathematicians or scientists; others have had a teacher or mentor who set them on the right track, and many are autodidacts. But I think it’s likely that they all have one thing in common: at an early age they discovered mathematics or physics or chemistry, the way a budding writer discovers words, the way Keats discovered Chapman’s Homer. This discovery not only revealed to them a new continent of the mind, it also served to insulate them from the deadening impact of a standard American education in math and science. By contrast, second-tier students have never been shown or discovered for themselves what math and science are really like, though they may have taken and even excelled in all the math and science courses that were offered in their schools. When these students get to college they experience a kind of culture shock. They discover that their mathematical and scientific education seems not to have fitted them for the serious study of math and science.

The process of killing a young person’s love of science and math often begins in grade school, with the teaching of arithmetic. The pedagogical principles that seem to inform conventional arithmetic instruction are identical with those that seem to have been adopted by the authors and publishers of best-selling freshmen calculus, physics, chemistry texts:

1. Focus on rote memorization of terms, definitions, formulas, rules, and algorithms.
2. Drill in solving stereotyped problems until students can do them in their sleep.

3. Sidestep questions relating to the provenance or justification of rules and algorithms. ("Why does this algorithm for long division work?" "It just does.")

4. Insist on the One Right Way to solve any given class of problems.

5. Eschew exams and homework assignments that require students to write complete English sentences and paragraphs.

Of course, some teachers downplay memorization and drills, explain rules and algorithms, use puzzles and games, encourage students to write, and even bring in a little history now and then. Nevertheless, it seems to be generally agreed that the central purpose of primary-school math instruction is to teach children to add, subtract, multiply, and divide integers, fractions, and decimals. Even educators who aren't content to stop with the teaching of rote skills believe you must start there. For example:

If we want students to feel comfortable with numbers, they need to have control of a certain body of facts, the very minimum of which would be the addition, subtraction, multiplication, and division facts up through, say, the 10's or 12's.


Now, as I am sure Professor Pagni knows, the "facts" he refers to aren't facts at all: they are theorems. There is a world of difference between a fact and a theorem. If you don't remember a theorem you can derive it; if you don't remember a fact you must look it up or ask someone. If you are alone on a desert island and don't recall the year in which the Congress of Vienna was convened, you're out of luck. But if you don't remember the value of 7x9, you can work it out — provided you understand (a) the meanings of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, (b) positional notation (that 234 means 2x100 + 3x10 + 4), and (c) the definition of the symbol x. The unique and (for all but the dullest students) exciting thing about arithmetic — once it is pointed out — is that all the properties of numbers — many of which are quite astonishing, and most of which remain to be discovered — are logical consequences of a handful of conventions and definitions. They can all be proven, and they are all "there," waiting to be discovered by the inquiring mind. Once bright students understand this remarkable fact, they are hooked. So long as they can continue to learn math in this spirit — the spirit in which it was created — they will stick with it and ask for more. Unfortunately, very, very few of them are offered that opportunity.

Some educators believe that students come to understand numbers by becoming proficient in doing sums. I don't agree. It is true that most people who are good at real math know the multiplication table at least up to 9x9, just as most good writers know how to type. But exercises in long division don't help one to understand numbers any more than typing exercises improve writing skill. Because cheap hand-held calculators are readily available (whereas machines that transcribe speech are still rare and expensive), being good at sums is actually less useful than being a fast typist. Someone who understands numbers but hasn't been drilled in long division may take much longer to divide one six figure number by another than someone who has learned and practiced the standard algorithm. But she will never make a mistake in the first figure or get the decimal point in the wrong place; and if she has forgotten the standard algorithm, she will be able to invent a new one.

But surely, you may say, a student cannot begin to appreciate the more sophisticated aspects of mathematics until she or he has memorized a sizable body of "facts" (even if you call them "theorems"). Not so. The very act of memorizing "number facts" inhibits understanding. The student who remembers that four times eight is thirty-two because she has had to work it out for herself a dozen times knows it better (in a qualitative sense) than the student who has simply memorized it. Students who are given enough interesting work with
numbers — and such work does exist, though it rarely finds its way into arithmetic workbooks — eventually acquire a degree of proficiency, just as writers who work at a keyboard acquire proficiency in typing.

The argument that memorization must precede understanding is flawed in another way. Few students who are forced to memorize arithmetic "facts" actually go on to form a deeper understanding of numbers. They go on to memorize new "facts" and algorithms — right through the freshman year of college if they have the stomach for it. Then, if they try to go further, they run into what many of them describe as a wall. It is a wall they have built for themselves, with expert guidance.

Learning to understand numbers is work of a very different kind from the mind-numbing tasks of memorizing definitions, rules, and algorithms and using them to solve stereotyped problems. It is challenging and stimulating, and it gives plenty of scope to invention and ingenuity. For example, consider the following familiar rule:

A number is divisible by 9 if and only if the sum of its digits is divisible by 9.

Is it true? A normally curious child will want to test the rule. She will invent examples, possibly in the hope of finding a counter-example. Sooner or later, perhaps with a little tactual help, she will notice that if the rule is true, then the same test for divisibility by 9 also applies to the sum of the digits: that is, the sum of the digits of the number you get when you add up the digits of the original number must itself be divisible by 9. So if the number you start with is divisible by 9, and if you keep summing digits, you eventually end up with the number 9 itself. Very mysterious. When this has sunk in, a curious child may — perhaps with some encouragement — ask herself why the rule works. If this question comes at an appropriate stage of mathematical development, she will eventually be able to answer it. And the answer leads to other questions and answers. For example, is there another number $n$ with the same property (that it is divisible by $n$ if and only if the sum of its digits is divisible by $n$)? What about numbers written in a system based on a number other than 10, the number 7, say? And so on. As you work on problems of this kind you can't help pick up some proficiency in adding, subtracting, multiplying, and dividing, just as people who write a lot eventually become pretty good typists. Much more important, though, you gain an understanding of the way numbers work, and you gain confidence in your ability to puzzle out their properties.

Not all children, perhaps, can develop a taste for working on problems like these; but more could develop such a taste than now get the chance. A good primary-school curriculum in arithmetic would allow these children to discover and cultivate the abilities and inclinations that underlie all mathematical and scientific thinking. It would set them on the right path. What would such a curriculum be like?

To begin with, unlike most present curricula, it would feature writing and discussion. Children would write and talk about puzzles and principles and practical problems. They would learn how algorithms are translated into the concrete mechanical language of counting boards and mechanical calculators, and how electronic calculators add, subtract, multiply, and divide, and how to translate their ideas into pictures and diagrams as well as words. They would learn the art of inexact arithmetic (estimation).

Arithmetic and more advanced branches of mathematics are nearly always taught ahistorically, as if they had always existed in their present form. This is a great mistake. A good arithmetic curriculum uses and teaches history. The world of numbers may be timeless, but our understanding of it is not. Knowing how key ideas came to be invented and how they influenced subsequent developments not only enlivens the study of arithmetic but leads to a deeper understanding of the ideas themselves. In particular, history is an invaluable guide to the real difficulties of mathematical and scientific concepts. For example, modern positional notation was invented less
than 2,000 years ago, in India, whence it spread to the Arab world and eventually to Europe. Yet its material analogue, the abacus, had been in use in many countries for centuries. What are the peculiar virtues of this notation? Why wasn’t it invented sooner? What role did the invention of zero play in the invention of positional notation? What does it mean to say that zero is an invention? Did zero have the same meaning for its inventors as it does today? How and why did the meaning change? And so on. Discussing and writing about such questions helps to develop an understanding of the number system that few students would otherwise develop.

But is this kind of understanding really necessary? If you will never need to understand algebra or still more advanced kinds of mathematics, it isn’t. Otherwise, it is. If you don’t thoroughly understand zero and the positive integers (but can merely manipulate them), you’ll understand negative numbers and fractions even less. Without a deep understanding of numbers and number system, you will never feel comfortable with algebra, and even less comfortable with analytic geometry. Of course, it’s possible to teach and learn algebra, analytic geometry, and elementary calculus as if they, too, consisted of definitions and algorithms to be applied mechanically to stereotyped problems. Indeed, that is how these subjects are usually taught and learned. But that kind of mathematical knowledge is not only useless and even disabling for the study of higher mathematics and science. It has very little value of any kind. It isn’t empowering in the way learning to speak and read a foreign language is empowering. The study of French opens many doors: it makes travel in French-speaking countries more interesting, allows you to understand the words of French songs, and makes it possible for you to read Proust or Flaubert or Astérix in the original. Mathematics is empowering in the same way: it opens up new areas of action and understanding. Newton invented the calculus to describe accelerated motion. Calculus is the language scientists use to describe all kinds of change. Without that language the greater part of modern technology would never have come into being. But for most students learning calculus means nothing more than memorizing meaningless rules to do meaningless exercises in differentiation and integration.

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Everything I’ve said about the teaching and learning of arithmetic applies to the teaching of physics and chemistry. In certain respects these subjects are extensions of mathematics. But they have another dimension as well: they describe regularities underlying natural phenomena, rather than regularities in the Platonic world of number. For this reason, demonstrations and experiments are an essential element in science education. Potentially they are the most interesting element. Math is an acquired taste, but every child likes to test the principles of mechanical equilibrium and to mix common household ingredients to see what happens. Unfortunately, children’s natural curiosity and playfulness aren’t always used to the best educational advantage. Laboratory exercises often seem designed to impact manipulative skills rather than understanding. Most of the lab manuals with which I am familiar read like cookbooks, and lab exercises in freshman physics and chemistry usually put a premium on the student’s ability to follow directions exactly. No wonder bright imaginative students tend to shun courses that feature labs of this kind!

Students do, however, need to achieve a concrete, practical understanding of the phenomena their physics and chemistry texts describe. If they’re learning about electrons, they need to see the track of an electron beam in a tube designed for that purpose, and they need to see what happens to the beam when they themselves twiddle the knobs that vary the strengths of the fields that accelerate and deflect it. They also need to become adept in moving back and forth between the theoretical and practical levels. Demonstrations and experiments that help students accomplish these goals need to be carefully coordinated — or better, integrated — with the theoretical curriculum.
Students also need plenty of opportunity to understand the equipment and procedures they're called upon to use. It isn't important for beginning students to become adept at the use of laboratory apparatus, any more than it is important for them to become adept at long division, but they do need to understand how the quantities that figure in the theories they are studying — quantities like mass, momentum, electric potential, entropy, and acidity — can actually be measured. Finally, a well-designed science curriculum would help students to gain an increasingly sophisticated understanding of the intricate relationship between theory and experiment in the physical sciences: a successful theory must meet the test of experiment, yet the interpretation of every experiment rests on a set of theoretical assumptions.

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If my criticism of conventional math and science education are valid, some radical reforms are called for. The teaching of foreign languages offers a useful paradigm for change. Until recently, most American students with three or four years of high-school French under their belts were unable to carry on a simple conversation in that language. By contrast, students in a good modern class hear and speak French from day-one. Within weeks they are conversing fluently, using a limited vocabulary and simple grammatical forms. Their grammatical knowledge and vocabulary grow in response to their expressive needs. The new approach to teaching foreign languages isn't just an improvement of the old approach. It has a different set of goals, and it uses different methods to achieve those goals.

Mathematics is also a kind of language (Galileo called it the language of nature). If we are to teach math and science more effectively than we do now, we need to formulate a new set of explicit goals and adopt methods for achieving these goals that are demonstrably effective. Such methods already exist; I have described some of their characteristics in the preceding paragraphs. Ideally, reform would begin in grade 1 and work its way upward, but those of us who teach math and science to college freshmen needn't and shouldn't wait for this to happen. Freshmen aren't too turned off to be turned on again. And when they are given greater scope for invention and ingenuity, and better opportunities to understand in depth, and to be genuinely challenged and stimulated — in other words, when they are allowed access to the real worlds of math and science — I bet we'll discover that the women among them are as eager and as able as the men.

Notes
1 Sheila Tobias, They're Not Dumb, They're Different. Research Corporation, 1990.
2 And, of course, more expensive.
3 Significantly, a similar decline in quality is not evident in textbooks and monographs addressed to more advanced undergraduates — textbooks in electromagnetic theory, quantum mechanics, or probability theory, for example. The best of these are better than their thirty-year old counterparts.
4 Tobias, They're Not Dumb.